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IMPLEMENTATION OF BARONE-ADESI AND WHEY MODEL IN FORCASTING FUTURES AND OPTIONS PRICE

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This study develops a method for estimating confidence intervals surrounding futures based forecasts of natural gas prices. The method utilizes the Barone-Adesi and Whaley model for option valuation to "back-out" the market's assessment of the annualized standard deviation of natural gas futures prices. The various implied standard deviations are then weighted and combined to form a single weighted implied standard deviation following the procedures outlined by Chiras and Manaster. This option implied weighted standard deviation is then tested against the more traditional "historical" measure the standard deviation. The paper then develops the procedure to transform the weighted standard deviation and futures price into a price range at the option expiration date. The accuracy of this forecast is then tested against 15 and 30 day average forecasts.

Keywords: option pricing, future price, predictive ability, price expectation.

Introduction

Although current futures prices provide a forecast of market participants' price expectations for future dates, the price alone provides no information regarding the distribution of these price expectations. To gain more information, the historical standard deviation of futures prices could be measured and used to develop a probability (or confidence interval) surrounding the current futures price. This method is used by many market participants but it may not be the best method to take advantage of all the information present in the market. An alternative method is to use information contained in options prices to develop a measure of the futures price standard deviation (Chiras & Manaster, 2005). Efficient option prices should contain all available information - including costly information that may not be available to the casual user - including the distribution of expectations of future prices. Options traders must combine their assessments of the historical standard deviation with perceptions of future price movements to accurately price the option under consideration. A properly specified option pricing model can be used to extract the traders' expectations about future price movements (Chiras & Manaster, 2005).

This paper will present a detailed discussion of the theory and intuition behind the pricing of options. This discussion is essential to understanding the option pricing models presented next and why the futures price standard deviation can be backed out of them. The predictive power of this weighted standard deviation is then compared - through regression analysis - to the predictive power of the historical standard deviation. The paper then presents the method for producing a futures/options based forecast and presents evidence that the mean derived from the futures price and option implied standard deviation is a superior predictor of the futures price at contract expiration.

Purpose of Study. This study develops a method for estimating confidence intervals surrounding futures based forecasts of natural gas prices. The central model uses options based measures of the distribution from which future natural gas prices will be drawn to develop probabilistic expectations about the mean and standard deviation of a gas futures contract at expiration (Barone & Whaley, 1997). These measures can help market participants (both financial and physical) develop realistic expectations about the range of possible price outcomes.

Theoretical Concept. Option Pricing Theory. Options on natural gas futures contracts were first actively traded in the early 1990’s. Although these options are fairly new, the market is quite robust. (Black, 2003) Natural gas futures options can now be purchased on contracts expiring several years in the future. Most of the trading action, however, is centered on contracts near expiration (one to six months until expiration).

Like all options, they give the owner the right but not the obligation to buy or sell a gas futures contract at a specified price (strike price). "call" option gives the owner the right to purchase the underlying commodity for the strike price at any time until the option's expiration date. Conversely, a "put" option allows the owner to sell a commodity at a specific price at any time until expiration (Black, 2006). If any of these options are not exercised, they will expire worthless.

As will be explained in detail, the main factors affecting prices of options on futures are the underlying price of the futures contract, the position of the strike price relative to the futures price, the risk free rate of interest, the time to the option’s expiration, and the volatility of the underlying commodity (Breeden & Litzenberger, 1998). Throughout this paper these variables will be represented as follows:

r risk free rate of interest.
σ. annualized volatility of the asset price
S underlying asset price.
t time remaining until the expiration of the option (years).
E exercise price of the option.

Therefore, the price of a call option is defined as:
$C = C(S,E,t,r,\sigma.)$

Only two relationships between the asset price and the exercise price can exist either $S>E$ or $S\leq E$. If the asset price is less than or equal to the exercise price the call option has no value and the owner will let it expire worthless.

When

$S\leq E, C(S,E,0) = \$0$

Alternatively, if the asset price is greater than the exercise price, the value of the option is the difference between the two values:

$S>E, C(S,E,0) = S-E.$
Combining the two scenarios:
$C(S,E,0) = \max(0,S-E).$

At expiration, the call option is worth the greater of 0 or S-E.

In an effort to set further boundaries on possible option prices, consider an option with an exercise price of zero and an infinite time horizon until expiration (Levy, 2005). The option can be exchanged at any time until expiration for the price of the asset. In other words the option can be exchanged at any time for the underlying asset itself. Because of this, the option's value is always equivalent to the value of the underlying asset.

$C(S,0,\infty) = S$

Thus the upper bound of the options price is the asset price itself, while as shown earlier, the lower bound is zero. All other options on this asset will fall between these two values.

Black-Scholes Option pricing Model. The first model to directly calculate options prices was developed by Fischer Black and Myron Scholes in their seminal work, "The Pricing of Options and Corporate Liabilities." Their model (B-S) values options on non-dividend paying stocks. Although we are ultimately looking for values (standard deviations) associated with American futures options, it is essential that we analyze the construction of the B-S model before we can analyze the special case of options.

One of the central assumptions of the B-S model is that the price movements of the underlying asset follow a stochastic 'wiener' process where asset prices change continuously through time and changes made over any given time period are distributed normally (Levy, 2005). B-S also makes the following simplifying assumptions: (1) there are no taxes or transaction costs; (2) the underlying asset exhibits no dividends or other leakage and its returns are lognormally distributed with constant variance; (3) markets operate continuously; (4) interest rates are constant and risk free. Black and Scholes derived their valuation model by forming a riskless hedged portfolio consisting of a long position in the underlying asset and a short position in the asset's call option (Grundy, 1999). The payoff to the hedged portfolio is the riskless rate of interest (in equilibrium) and represents a nonstochastic partial differential equation for the value of the asset call option. The partial differential is expressed as:

(1) $\partial Ce/\partial t = rCe - .5(\sigma^2 S^2 \partial^2 Ce/\partial S^2)$

Ce = call option value and can be solved subject to the following boundary conditions:

(2) $Ce(S,E,t) = MAX\ [0,S-E]$ where $t=0$

(3) $Ce(S,E,t) = 0$ where $S=0$

Formally the Black-Scholes model for a call option is as follows:

$Ce\ (S,E,r,t,\sigma) = SN(d1) - Ee^{-rt}N(d2)$
where $d1 = [\ln(S/E)+(r+.5\sigma^2)t]/\sigma\sqrt{t}$
$d2 = d1 - \sigma\sqrt{t}$

N(d1) and N(d2) = cumulative normal probabilistic values of d1 and d2. The use of the normal probability function gives the B-S model its ability to incorporate the price risk of the asset into the option price (Latane, 2006).

Using the values from the previous intuitive example where:

S = \$2.00 E = \$2.00
t = 1 year r = 6% ;
σ.= 5%

The price of the option can be calculated using the B-S model:

$d1 = [\ln(2/2) + (.06+.5(.05)^2)1]/.05$
$= 1.225$
$d2 = 1.225 - .05$
$= 1.175$

d1 and d2 are simple z scores that can be looked up in a table:

N(1.225) = .8897
N(1.175) = .88

Therefore $C = 2(.8897) - 2e^{-(.06)}(.88) = \$.122$

This value, when multiplied by 50 becomes \$6.10. This is quite close to the \$5.66 value derived from our portfolios 1 and 2. Note that the term Ee-rt is the present value of the exercise price with continuous discounting. The B-S model essentially becomes:

C = SN(d1) - PV(E)N(d2)

Taking this one step further, if the stock had no risk, N(d1) and N(d2) would both equal one. The B-S equation then, would further be simplified to C = S - PV(E) , the exact equation we found through intuition.

Pricing Options on Futures - Black Model Like asset options, options on futures contracts give the owner the right to exercise the option and give the seller the obligation to perform on the contract (Black, 2003). As noted earlier, the B-S model was developed to price options on non-dividend stocks. Options on assets without dividends have the same price as the equivalent option but options on futures contracts do not fall into this category. Because these contracts are settled on a daily basis, the cash flows associated with settlement act as a continuous dividend. Thus, futures contracts violate one of the B-S assumptions.

In order to help solve the problem, Black (1976) adjusted the B-S model to value call options on futures contracts:

C (F,E,t,r,σ) = e-rt[FN(d1*)-EN(d2*)]

Where t = time to expiration of the forward contract.
r = risk free rate of interest
F = current futures price for contract expiring at t
E = strike price. σ.= annualized standard deviation of the futures contract price.

d1* =[ln(F/E)+.5σ2t]/σ√t

d2* = d1* - σ√t

Notice that in the Black model the r term drops out of the calculation of d1 (Black, 2003). Also, the entire pricing equation is discounted by e-rt. Therefore under the certainty assumption, the option is worth the present value of the proceeds:

C = e-rt[F-E]

Early exercise is possible and often desirable when dealing with options on futures contracts. When the option is exercised you receive the explicit value from that option (F-E), but give up the right to any future gains above F-E. Consider the case of a call option with a strike price of \$1.50 on a futures contract currently selling for \$2. Here the trader could exercise the option and collect \$.50. This \$.50 can then earn interest through the original option expiration date. The interest accrued is equal to ert(F-E)-(FE).

These calculations only hold when the futures price does not change over the remaining life of the contract - an unlikely scenario. Instead there is a chance that the futures price may move up and the trader will lose additional profits. This tradeoff between exercising early and foregoing potential additional gains is what makes options difficult to model.

(Gerundy,1999), Values of N(d1*) and N(d2*) approach one when the futures price becomes very large relative to the exercise price{ C = e-rt[F0,t-E]}. As shown above, the minimum value for a European futures option is e-rt[F-E]. This happens when it is almost certain that the option will remain in the money and pay F-E at expiration. Basically, at high futures prices the option value converges to the present value of the exercisable proceeds. It will not exceed this value because the proceeds are not available until the expiration date.

Table 1: **Implied Standard Deviations and Vega's Indonesia Stock exchange closing data August 29, 2008**
Contract price being forecasted: October 2008 Natural Gas
Futures contract value: \$2.714
Option expiration date: Sept. 25, 2008
Time to expiration: .0742years.
Short term interest rate (bond yield): .0524
Weighted Implied Standard Deviation: .592 (.161 over t+T)

Put/Call	Volume	Strike Price	Option Value	Implied Std. Dev.	Vega
C	11	\$2.00	0.723	0.300	0.000
C	10	\$2.20	0.531	0.597	0.107
C	5	\$2.25	0.486	0.586	0.130
C	6	\$2.35	0.401	0.574	0.180
C	7	\$2.40	0.362	0.573	0.205
C	5	\$2.50	0.287	0.559	0.250
C	42	\$2.60	0.226	0.567	0.281
C	8	\$2.65	0.199	0.569	0.284
C	306	\$2.70	0.174	0.569	0.292

C	191	$2.75	0.148	0.558	0.295
C	2438	$2.80	0.129	0.563	0.292
C	131	$2.85	0.111	0.565	0.283
C	158	$2.90	0.096	0.570	0.269
C	197	$3.00	0.073	0.585	0.253
C	40	$3.05	0.064	0.595	0.231
C	27	$3.10	0.055	0.598	0.233
C	2027	$3.20	0.039	0.599	0.183
P	5	$2.05	0.012	0.678	0.085
P	35	$2.20	0.019	0.604	0.108
P	302	$2.30	0.029	0.574	0.155
P	201	$2.35	0.038	0.574	0.180
P	12	$2.40	0.049	0.574	0.205
P	27	$2.50	0.074	0.561	0.250
P	145	$2.65	0.135	0.568	0.284
P	485	$2.70	0.16	0.569	0.292

Table 2. Weighted Implied Standard Deviations

Put/ Call	j	Strike Price Table 2	σφ : Ωεγη	Vj(sj/Wj) ted Impl	Σς(σ/Ω)	**ied Standard Deviatiosj· [Vj(sj/Wj)/ S V(s/W)]**	**ns Annualized WISD**
C	1	2.00	0.300	0.000	39.331	0.000	0.592
C	2	2.20	0.597	0.121	39.331	0.002	0.592
C	3	2.25	0.586	0.157	39.331	0.002	0.592
C	4	2.35	0.574	0.257	39.331	0.004	0.592
C	5	2.40	0.573	0.324	39.331	0.005	0.592
C	6	2.50	0.559	0.487	39.331	0.007	0.592
C	7	2.60	0.567	0.705	39.331	0.010	0.592
C	8	2.65	0.569	0.811	39.331	0.012	0.592
C	9	2.70	0.569	0.956	39.331	0.014	0.592
C	10	2.75	0.558	1.112	39.331	0.016	0.592
C	11	2.80	0.563	1.274	39.331	0.018	0.592
C	12	2.85	0.565	1.440	39.331	0.021	0.592
C	13	2.90	0.570	1.600	39.331	0.023	0.592
C	14	3.00	0.585	2.025	39.331	0.030	0.592
C	15	3.05	0.595	2.147	39.331	0.032	0.592
C	16	3.10	0.598	2.532	39.331	0.039	0.592
C	17	3.20	0.599	2.806	39.331	0.043	0.592
P	18	2.05	0.678	4.823	39.331	0.083	0.592
P	19	2.20	0.604	3.437	39.331	0.053	0.592
P	20	2.30	0.574	3.069	39.331	0.045	0.592
P	21	2.35	0.574	2.718	39.331	0.040	0.592
P	22	2.40	0.574	2.400	39.331	0.035	0.592
P	23	2.50	0.561	1.894	39.331	0.027	0.592
P	24	2.65	0.568	1.195	39.331	0.017	0.592
P	25	2.70	0.569	1.039	39.331	0.015	0.592

The standard deviation of future logarithmic October futures prices implied by the options is:

$$(WISD) \bullet \sqrt{T} = .592\sqrt{(.0742)} = .161$$

with a mean of:

$$ln(F) - \sigma 2T/2 = ln(2.714) - .026/2 = .985$$

Testing the WISD. If the option market is truly efficient the prices will reflect all available information. Additionally, the variances derived from the pricing model should reflect all information contained in the history of the futures price as well as any additional information that may have been available at the time of the trade (Deaves, 1992). Thus the WISD values obtained from options prices may reflect future values of the standard deviation better than historical observations alone.

The following test is designed to determine some of the predictive characteristics of the information contained in the natural gas futures options prices. The test follows the methodology used by Chiras and Manaster (2005) to determine the predictive ability of stock option prices. The hypothesis behind the test is that standard deviations inferred by options prices have been better predictors of future standard deviations than standard deviations obtained from historic futures prices.

The test involves the creation of three monthly series of annualized volatility measures covering natural gas data from1993-1996. Like the previous WISD example, all of the options and futures data used in this test were obtained directly from the Indonesian Stock Exchange. The yield data was obtained from the Federal Reserve (MacMillan, 2006). At the beginning of each month (or as close to the beginning as possible), a historical (SDHIST), weighted option implied standard deviation (WISD), and a future (SDFUT) standard deviation are created.

With the beginning of the month designated day t, the annualized historical standard deviation is measured with futures price movements from t-14 up to and including t (15 observations). To annualize the data, the log of each daily price change is calculated (ln[t-14/t-13]). This new series is then averaged and differenced from the mean. The sum of these differences is then divided by the number of elements (in this case fourteen) and multiplied by the number of trading days in a year. The square root of this final number is the annualized standard deviation. The future standard deviation is calculated in a similar fashion except the price movements are measured from time t+1 until the expiration of the futures contract (Cox, 1996). This may be anywhere from 12 to 17 observations.

Finally, the WISD is calculated from closing options prices and the closing futures price on day t. Like the previous WISD example, any option with a volume less than five is discarded. Because of the low volume, the prices of these options may not be good indicators of market conditions. In addition, any option, put or call, with a strike price more than 25% away from the observed futures price is removed.

Forty nine observations for each element (SDFUT, SDHIST, and WISD) were calculated using the data set. The 49th observation was calculated since the set actually begins with November 1992 options data on the December 1992 futures contract. Using regression analysis, the SDHIST's and WISD's are then compared to the SDFUT's to determine which predictor is superior over the time period. The following two regression equations are used to test the hypothesis:

$$SDFUT = ah + BhSDHIST + eh$$
$$SDFUT = ao + BoWISD + eo$$

a_x and B_x are the coefficients on the constants and independent variables respectively ex represents the error of each regression. Table 3 displays the regression output from equation 18 and table 4 displays the output from equation 19.

Both regressions indicate that the SDHIST's and WISD's may be significant. Their high t-ratios indicate that we can reject the null hypothesis that the B coefficient is zero for each regression. The most useful information that can be obtained from the regressions is the adjusted R-squared values. The R-squared value for equation 18 is .507, indicating that historic standard deviations (and the constant) explain 51% of the future standard deviations of futures price movements (Latane, 2006). On the other hand, the R-squared for equation 19 is .60. This implies that the option implied standard deviation can explain 60% of the future standard deviations. Also note that the constant in the regression of equation 19 is not significant. This indicates that the R-squared does not significantly improve with the inclusion of a constant.

The differences between the two equations are not dramatic. However, based on the evidence provided by the two R-squared values, the WISD based equation represents roughly a 20% improvement in predictive power over the SDHIST's. Based on this evidence we can conclude that the WISD based predictions are better indicators of future standard deviations of futures price movements.

Constructing the Options Based Forecast Arbitrarily constructing a price interval around the observed futures price (say ± 25%) one can calculate the probability that the futures price will fall within the constructed range at option expiration. Following the example from section 9, the ±25 □□interval around the October futures price becomes $2.036 to $3.39 MMBtu. The probability is expressed as:

$$\text{Prob } (2.036 < F < 3.39)$$

Since the mean and standard deviation produced by the model are expressed as logs equation should be expressed as:

$$\text{Prob } (\ln(2.036) < \ln(\text{futures}) < \ln(3.39))$$

Expressing in standardized normal form:

$$\text{Prob } [(.71-.985)/.16 \leq (\ln(\text{futures})-.985)/.16 \leq (1.22-.985)/.16].$$

Or $\text{Prob}(-1.7 \leq z \leq 1.47)$

Checking a normal distribution table, the cumulative probability that the final futures price will be less than $2.036 is .044 while the probability gas prices will be below $3.39 is .929, thus the probability that October spot gas prices will fall between $2.036 and $3.39 is 88%.

Predictive Ability of the Options Based Forecast. Although the options based forecast model is most useful when the forecasted mean is combined with the qualitative information of the implied standard deviation, it is also useful to know how accurate the model is at predicting the exact level of a futures price at the expiration of the contract (Leahy 2006). To test the model's predictive ability, a series of monthly forecasts of the mean were constructed using the WISD methodology and equation 15 (e raised to the calculated mean of 17). The monthly forecasts were then compared to the actual values of the gas prices on the dates being forecast. The predicted and actual prices for each forecast date are listed in Appendix Table 1a. Each forecast was derived from mid-month options prices. Again, all options with less than five trades on the forecast date were discarded and any option with a strike price of more than □25% of the underlying futures price was also removed from the test. For comparison purposes, predictions based on a 15-day and 30-day moving averages of the futures price were also calculated.2 The sum of squared errors, mean square error, and root mean square error for each of the three forecasting methods.

Over 42 observations, the standard deviation of the error produced by the options estimates was .28 with a maximum underestimate of $1.088 and a maximum overestimate of $.55. This compares to a standard error of .33 for the 15-day average and. 37 for the 30-day average. The evidence indicates that the options based forecast is superior to either of the moving average based prediction methods.

Conclusions

This paper demonstrated that information about the future distribution of natural gas prices can be obtained from the prices of options on gas futures contracts. The theory behind option pricing was outlined. A model for valuing options on futures contracts was presented and linked to the theory. Finally a method for using the pricing model to derive and weight the implied standard deviations contained in option prices was discussed. Since the model depends on futures and options prices, it theoretically contains all the information utilized by the open market in pricing the futures and options prices themselves. If these markets are in fact informational efficient, the mean and standard deviation produced by this model should be an assessment of the market's 'consensus' opinion of their future values.

Regressions run on the weighted implied standard deviations indicated that they may be better estimators of the future standard deviations than ordinary standard deviations based on historical information (Latane, 2006). Additionally, the ability of the futures/options derived mean to predict the expiration price of the futures contract was explored. The futures/options based forecast compared favorably to the 15-day and 30-day rolling average estimates.

Market analysts can use the methods outlined here to benefit from expert opinion and expensive information often associated with market professionals and complex models without actually hiring consultants or paying for expensive market forecasts (Levy, 2005). With this in mind, analysts can use this model to develop new forecasts based entirely on Indonesian Stock exchange data and model output. They could also assess the probability of prices developed using old forecasting methods or independently verify and critique external forecasts (such as those purchased from financial service consultants).

The method of deriving market based forecasts outlined in this paper is easy to implement and quite flexible. Many computer programs exist (often as add-ins to popular spreadsheet software titles) that will solve for the implied standard deviation of an option given the current option price, futures price, risk-free interest rate and time until expiration. The user will not have to bother with the onerous task of coding the solution to the B-AW model. In addition, several financial services companies now supply real-time trading data via computer. Generally, this data can be linked directly to a spreadsheet allowing the user to track changes in market prices and conditions (Chiras, 2005). This real-time data coupled with a spreadsheet option valuation model could allow any user to monitor instantaneous changes in the probabilities surrounding several months of futures forecasts.

The Indonesian Stock Exchange currently lists options on contracts with maturities of up to three years, however, the practical range of the model is limited by the low volume of trades that actually take place on the long-range options. An observation of volumes indicates that the practical limit on forecasts is about four months. The market for natural gas futures contracts has, however, been growing quickly over the past few years. This rapid growth may carry-over into the market for options on these contracts. As the market for longer term options increases, the reliability of long range option based forecasts will also improve.

REFERENCES

Barone-Adesi, G., and R. E. Whaley (1997). "Efficient Analytic Approximation of American Option Values." *Journal of Finance*, 42, pp. 301-302.

Black, F., M. Scholes (2003). "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* (May), pp.637-659.

Black, F. 2006). "The Pricing of Commodity Contracts." *Journal of Financial Economics*, 3, pp. 167-179.

Breeden, D. T., and R. H. Litzenberger (1998). "Prices of State-Contingent Claims Implicit in Options Prices." *Journal of Business*, Vol. 51, No. 4, pp.621-651.

Chiras, D.P., and S. Manaster (2005). "The Information Content of Option Prices and a Test of Market Efficiency." *Journal of Financial Economics*, 6, pp. 213-234.

Cox, J. C., and S. Ross (1996). "The Valuation of Options for Alternative Stochastic Process." *Journal of Financial Economics*, 3, pp. 145-166.

Deaves, R., and I. Krinsky (1992). "Risk Premiums and Efficiency in the Market for Crude Oil Futures." *Energy Journal*, Vol. 13, No. 2, pp. 93-118.

Grundy, B. D. (1999). "Option Prices and the Underlying Asset's Return Distribution." *Journal of Finance*, Vol. 46, no. 3, pp. 1045-1069.

Jarrow, R., and A. Rudd (2002). "Approximate Option Valuation for Arbitrary Stochastic Process." *Journal of Financial Economics*, 10, pp. 347-369.

Latane, H. A., and R. Rendleman, Jr. (2006). "Standard Deviations of Stock Price Ratios Implied in Option Prices." *Journal of Finance*, Vol. 31, no 2, pp. 369-381.

Leahy, M. P. and C. P. Thomas (2006). "The Sovereignty Option: The Quebec Referendum and Market Views on the Canadian Dollar." Board of Governors of the Federal Reserve System International Finance Discussion Papers, No. 555.

Levy, H. (2005). "Upper and Lower Bounds of Put and Call Option Value: Stochastic Dominance Approach." *Journal of Finance*, Vol. 40, no. 4, pp. 1197-1217.

MacMillan, L. W. (2006). "Analytic Approximation for the American Put Option." *Advances in Futures and Options Research*, 1A, pp.119-139.